Optimal Shape Design of Dielectric Structure Using FDTD and Topology Optimization

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Abstract — **In this paper, an optimal design method based on the FDTD technique and the topology optimization is proposed. Topology Optimization is a scheme to search an optimal shape by adjusting the material properties of design space. And by introducing the adjoint variable method, we can effectively estimate a derivative of objective function with respect to design variable. In order to verify our method, a shape design problem of dielectric structure is tested in the TM^z case. In this example, the permittivity density at each grid is taken as design variable.**

I. Introduction

The conventional shape design method is to get the optimized performances by changing the shape of design space [1]. Therefore the position or parametric vectors are taken as the design variables which specify or control the interface surface between regions. Also the unstructured mesh generator is needed because the shape of geometry is changed at each design iteration. And if the quality of unstructured grids is poor, a large design error may be generated. And also, there is a limit on the geometrical topology of the model.

Recently, a new design method which was called Topology Optimization was proposed in the field of the structural engineering by Bensoe et al [2][3]. They used a large number of adjacent grids in which the material properties are allowed to vary continuously between solid and void. By determining that each grid is solid or void so that design criteria are satisfied, one can obtain the optimal design shape. This method does not require an initial shape of design space. Therefore, it is possible to obtain an arbitrary topology of design space. In the mid-90's, Dyck introduced the Optimal Material Distribution (OMD) method to the optimal design of the magnetic systems [4]. And also, H.B. Lee used this method to obtain a matched load in the waveguide structure [5].

In this paper, we applied the Topology Optimization method to the optimal shape design problem in a wideband. The FDTD technique is one of the methods widely used to treat transient electromagnetic waves [6]. However, the design sensitivity cannot be derived in a straightforward way from the only FDTD method. So, using the adjoint equation derived from the FETD method, one can transform it into the Maxwellian coupled equations [7]. These equations can be solved by the FDTD method.

As a design example, a dielectric structure was designed so that it focuses a normal incident plane wave at a specific point. The initial design space was uniformly filled with dielectric material. And the permittivity density of each grid is taken as design variable. In order to illuminate a plane wave, we adopted total-field/scattered-field (TF/SF) scheme. And in order to reduce the computational domain, a PML technique is adopted.

II. Topology Optimization

In the topology optimization, the entire design space to be optimized is divided into small grids and the material composition of each grid is taken as design variable. By controlling the material composition of each grid, the optimized shape can be constructed approximately. The key concept of this method is how to treat the material composition in order to estimate the objective function, and obtain the final shape of device. There are two methods to treat the material composition, the homogenization method and density method. Homogenization method provided solid material physical and mathematical basis for the calculation of the material properties of the composite, or intermediate materials. On the other hand, the density method takes the material density of each grid as the design variable and it does not concern the microstructure but the results only. In this paper, the density method is preferred.

To apply the density method to the optimization scheme, the normalized density of material p is introduced, which takes the value 0 and 1. Using the normalized density p, one can represent the relative permittivity as

$$\varepsilon_r = 1 + (\varepsilon_{ro} - 1)p^h \tag{1}$$

where ε_{ro} is the maximal relative permittivity, and h is the exponent which defines the relationship between material property and normalized density. When p is 0 in a specific grid, it means that the grid is empty. And when p is 1, it means that the grid is entirely filled with dielectric material with $\varepsilon_r = \varepsilon_{ro}$. The value of p between 0 and 1 corresponds to the intermediate density which is to be converged by adjusting h. The plot of ε_r and p is shown in Fig. 1.

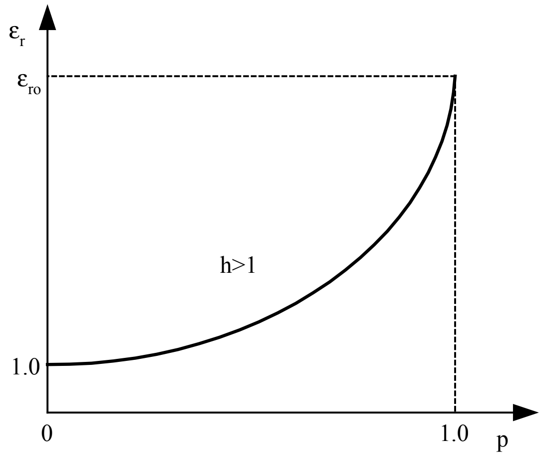


Fig. 1. Relationship between normalized density and permittivity. Usually, the value of h is chosen between 2 and 4 considering the convergence rate.

III. Design Sensitivity Analysis

From Maxwell's equation, the 2-D TM^z scalar wave equation is written as

$$\nabla \cdot \left(\frac{1}{\mu_r} \nabla E_z \right) - \frac{\varepsilon_r}{c_0^2} \frac{\partial^2}{\partial t^2} E_z = \mu_0 \frac{\partial}{\partial t} J_z \qquad (2)$$

where ε_r denotes relative permittivity and c_0 means velocity of light. Using the variational form and Galerkin's formula, one can discretize the scalar wave equation and construct the matrix equation as following

$$[M]\{\ddot{E}_z\} + [K]\{E_z\} = \{Q\} \qquad (3)$$

subject to

$$E_z(0) = 0,\ \dot{E}_z(0) = 0 \qquad (4)$$

where dot denotes the time derivative. Matrices $[M]$, $[K]$ and $\{Q\}$ can be represented as

$$M_{ij}^e = \frac{1}{c_0^2} \int_{\Omega_e} \varepsilon_r^e N_i N_j d\Omega_e \qquad (5a)$$

$$K_{ij}^e = \int_{\Omega_e} \nabla N_i \cdot \nabla N_j d\Omega_e \qquad (5b)$$

$$Q_i^e = -\frac{1}{\mu_0} \int_{\Omega_e} N_i \dot{J}_z^e d\Omega_e \qquad (5c)$$

where N_i is a scalar basis function and the superscript 'e' is the abbreviation of element.

An objective function F for estimating a response is defined as

$$F(E_z, p) = \int_{\Omega_m}\int_0^{T_f} G(E_z(t), p)\, dt d\Omega_m \qquad (6)$$

where T_f is a fixed final time and G is an arbitrary differentiable function of E_z and p. Ω_m is the observation domain where the electric field E_z is measured. The design variable p is the normalized density variable defined in the design space. The design sensitivity is obtained from the total derivative of F with respect to p as

$$\frac{dF}{dp} = \iint_{\Omega_m} \left(\frac{\partial G}{\partial p} + \frac{\partial G}{\partial E_z} \frac{\partial E_z}{\partial p} \right) dt d\Omega_m . \qquad (7)$$

Using the adjoint variable vector λ, one can derive the adjoint equation of (2) [5] as following

$$[M]\{\ddot{\lambda}\} + [K]\{\lambda\} = \left\{ \frac{\partial G}{\partial E_z} \right\}^T \qquad (8)$$

subject to

$$\lambda(T_f) = 0,\ \dot{\lambda}(T_f) = 0 . \qquad (9)$$

Equation (9) is a terminal condition on λ for solving (8). To deal with terminal conditions, the backward time scheme, $\tau \equiv T_f - t$, is introduced. Then (8) can be converted into the initial-value problem. Using (7) and (8), one can transform the design sensitivity (7) into

$$\frac{dF}{dp} = \int_{\Omega_m}\int_0^{T_f} \left(\lambda^T(t) \frac{\partial R(p,t)}{\partial p} + \frac{\partial G(E_z, p)}{\partial p} \right) dt d\Omega_m \qquad (10)$$

where

$$R(p,t) = \{Q(t)\} - [M]\{\ddot{\tilde{E}}_z(t)\} - [K]\{\tilde{E}_z(t)\}. \qquad (11)$$

The notation ~ indicates that argument is held constant for the derivative process with respect to p. However, since the design variable p is only dependent on the material property ε_r, $[M]$ is only dependent on ε_r, therefore,

$$\frac{\partial R}{\partial p} = -\left[\frac{1}{c_0^2} \int_{\Omega^e} \frac{\partial \varepsilon_r^e}{\partial p} N_i N_j d\Omega_e \right] \{\ddot{\tilde{e}}_z\}. \qquad (12)$$

From (1), a derivative of ε_r with respect to p is

$$\frac{\partial \varepsilon_r^e}{\partial p} = h(\varepsilon_{ro} - 1) p^{h-1} \qquad (13)$$

From the uniqueness theorem, (8) can be transformed into the corresponding Maxwellian coupled equations [6] as

$$\frac{\partial \lambda_x^B}{\partial t} = -\frac{\partial \lambda_z^E}{\partial y} \qquad (14a)$$

$$\frac{\partial \lambda_y^B}{\partial t} = \frac{\partial \lambda_z^E}{\partial x} \qquad (14b)$$

$$\frac{\partial \lambda_z^D}{\partial t} = \frac{\partial \lambda_y^H}{\partial x} - \frac{\partial \lambda_x^H}{\partial y} - J_z^\lambda \qquad (14c)$$

subject to

$$\lambda_z^E(T_f) = \lambda_x^H(T_f) = \lambda_y^H(T_f) = 0 \qquad (15)$$

And these adjoint variable vectors satisfy the constitutive relation same as the electromagnetic field vectors. That is,

$$\vec{\lambda}^D = \varepsilon \vec{\lambda}^E \ ,\ \vec{\lambda}^B = \mu \vec{\lambda}^H \qquad (16)$$

In (14c), J_z^λ is a pseudo electric current density and can be obtained from the relation of

$$\left.\frac{\partial G}{\partial E_z}\right|_{\Omega_m} = \mu_0 \int_{\Omega_m} N_i \dot{J}_z^\lambda d\Omega\,. \qquad (17)$$

For a specific point in design domain, J_z^λ can be represented as a point current source following

$$J_z^\lambda(x,y,t) = J_z^\lambda(t)\delta(x-x_m, y-y_m) \qquad (18)$$

By inserting (18) into (17) and assuming that the grid is a regular quadrilateral, the right-hand side of (17) is

$$\mu_0 \int_{\Omega_m} N_i \dot{J}_z^\lambda d\Omega = \mu_0 \dot{J}_z^\lambda(t)\Delta \qquad (19)$$

where Δ is the area of grid. Therefore, J_z^λ can be rewritten as

$$J_z^\lambda(t) = \frac{1}{\mu_0 \Delta}\int_0^t \frac{\partial G(t')}{\partial E_z} dt' \qquad (20)$$

In order to estimate the spectral density of an electric field in design domain, in this paper, we define G as

$$G(t) = \frac{1}{2}\left(E_z(t)\right)^2 \qquad (21)$$

Then, (14a)-(14c) can be also solved by using FDTD technique with terminal conditions (16). And introducing the electric fields and adjoint variables solved by using FDTD solver to (10) and (12), one can calculate the design sensitivity also. As an optimization algorithm, the steepest descent method is used.

IV. Numerical Examples

We applied the proposed method to the design of a dielectric lens in 2-D TM^z case. The analysis model is shown in Fig. 2. In order to realize a plane wave source, the total-field/scattered-field scheme was adopted. The incident wave is a gaussian modulated by a sine function with center frequency of 10GHz. The design object is to maximize the electric field energy over a wideband at the focal point when the pulsed TM^z plane wave is normally incident along the x-axis. The normalized material density of each grid is taken as design variable *p*. By adjusting *p*, we can find the optimal shape. The objective function is defined as

$$F = \int_0^{T_f} \left(E_z(t)\big|_f\right)^2 dt \qquad (22)$$

where $E_z(t)|_f$ is the electric field at the focal point. Fig. 3 shows the gray-scaled density at various iterations. The white-colored grid means that it is empty. The black-colored grid is completely filled with dielectric material. And the gray-colored grid has an intermediate density. The initial design space was filled with uniform dielectric material as shown in Fig. 3(a). We can see that the dielectric guide structure is constructed gradually. At the 10th iteration, one can see that many of grids are the state of intermediate density. However, as the design step is progressing, one can find that most of gray grids are eliminated as Fig. 3(c). And Fig. 4 shows the convergence rate of the objective function *F*. After the 10th step, the value of objective function increases slowly. It is because the elimination process of intermediate density mainly occurs without the great change of topology.

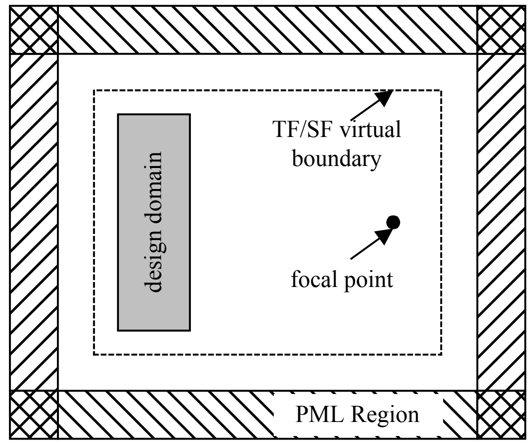
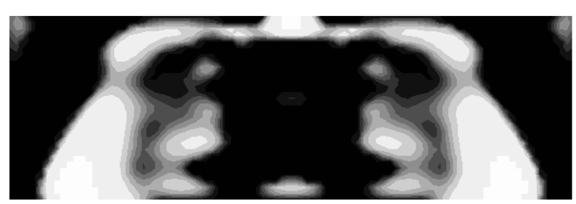


Fig. 2. Design model configuration. The TM^z plane wave is normally incident to the total-field region along the x-axis and the PML region with 8 layers is located at the outer boundaries. Size of design domain is 30 by 90.

(a) 0 iteration

(b) 10 iterations



(c) 90 iterations

Fig. 3. The material distribution in the design space at various iterations. In this model, we set ε_{ro} =2.7.

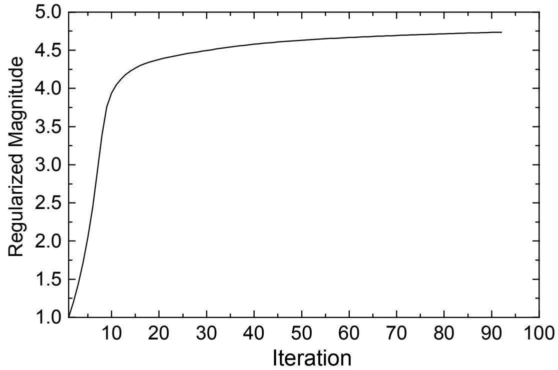


Fig. 4. The convergence rate of regularized objective function with the initial value of *F*. After the 10th iteration, the increase of objective function slows down.

Fig. 5 shows the amplitude of electric fields at the focal point in the frequency domain at various iterations. As expected, the magnitude of electric field intensity of the optimized model is greater than that of the initial model in a wideband.

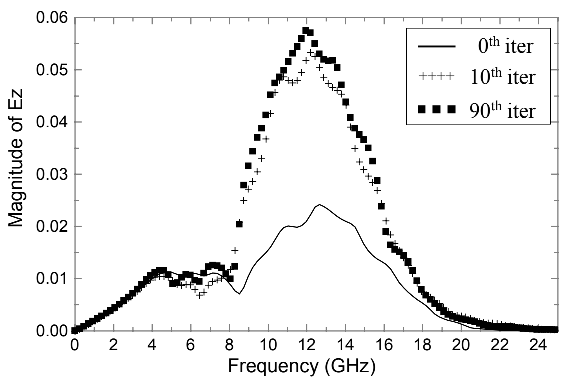


Fig. 5. FFT results of electric field at various iterations.

V. Conclusion

A new optimal design method using FDTD method and Topology Optimization has been proposed. Using the Topology Optimization technique, one can design an arbitrary topology of model regardless of initial shape. The proposed design algorithm has been verified by designing a dielectric structure to maximally focus the energy of electric field at focal point.

References

[1] E. J. Haug, K. K. Choi and V. Komkov, *Design Sensitivity Analysis of Structural System*, New York: Academic Press, 1986.

[2] M. P. Bendsoe and N. Kikuchi, "Generating optimal topologies in structural design using a homogenization method," *Computer Methods in Applied Mechanics and Engineering*, vol. 71, pp. 197-224, 1988.

[3] M. P. Bendsoe, "Optimal shape design as a material distribution problem," *Structural Optimization*, vol. 1, pp. 193-202, 1989.

[4] D. N. Dyck and D. A. Lawther, "Automated design of magnetic devices by optimizing material distribution," *IEEE Trans. Magnetics*, vol. 32, pp. 1188-1193, 1996.

[5] Hong-bae Lee, *Computer Aided Optimal Design Methods for Waveguide Structures*, Ph.D. Dissertation, Seoul National University, 1995.

[6] Y. S. Chung, C. Cheon, I. H. Park and S. Y. Hahn, "Optimal Design Method for Microwave Device Using FDTD and Design Sensitivity Analysis," IEEE *Trans. MTT.*, vol. 48, pp. 2289-2296, 2000.

[7] Allen Taflove, *Computational Electrodynamics: The Finite-Difference Time-Domain Method*, Boston: Artech House, 1995.

[8] J. L. Volakis, A. Chatterjee and L. C. Kempel, *Finite Element Method for Electromagnetics*, New York: IEEE Press, 1998.